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One-Loop Corrections To 4-Graviton Interaction In SST II and Heterotic String Theory

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Abstract

Various one-loop corrections to the four-graviton interaction in the SST II and the heterotic string theory are sorted out and some of them explicitly calculated. In particular the leading for $\alpha' \to 0$ correction from the heterotic amplitude is obtained. Also the non-leading non-analytic corrections are discussed and their correspondence to the one-loop supergravity diagrams is used for their classification

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I. Introduction

Much effort has been recently devoted to obtaining the low energy effective lagrangians in string theories. One approach consists in calculating the string scattering amplitudes for massless particles in the tree approximation and then writing down an effective lagrangian which reproduces those amplitudes order by order in the expansion in $\alpha' P^2$ (P is a typical momentum of the external massless particles). This systematic decoupling of heavy modes generates an effective lagrangian with an infinite number of local effective vertices (4-point, 5-point, etc. and with higher and higher number of derivatives) which, for the gravity sector, are higher curvature terms and higher derivative interactions.

It is of some interest to discuss quantum string corrections to the low energy effective action for massless modes and preliminary results on one-loop corrections in the SST II and in the heterotic string theory have been reported.[1,2]

In this paper we extend those results in two directions. In the second section we discuss more systematically various one-loop string corrections to the effective low energy 4-graviton interaction in the SST II. As expected we identify two types of corrections: One which renormalize the local vertices already present at the tree level and the others-genuinly non-local corrections reflecting the presence of the branch cut singularities at massless thresholds of the one-loop amplitude. The first category splits further into renormalization by heavy modes running in the loop and renormalization by part of the massless loop contribution. In the third section we calculate the leading one-loop correction to the 4-graviton interaction in the heterotic string theory and discuss briefly the presence of further corrections similar to those discussed in detail for the SST II. Finally we comment on the correspondence between the calculated one-loop string corrections to the low energy 4-graviton interaction and the one-loop corrections in the quantum field theory (with $1/\alpha'$ cut-off) with the lagrangian given by the low energy effective lagrangian read off from the tree level string amplitudes (with higher curvature and higher derivative terms).

II. One-Loop Corrections To The 4-Graviton Interaction in the SST II.

The one-loop four-graviton amplitude in the SST II reads (see e.g. ref.[1]) ($\kappa^2 = 4g^2(\alpha')^4, d^2z_i = 2d(Rez_i)d(Imz_i), d^2\tau = 2d(Re\tau)d(Im\tau)$):

$$\begin{aligned} A(K_1,\cdots K_4) &= \frac{\kappa^4}{2^{12}\pi^6}\frac{1}{\alpha'}\xi^1_{\mu_1\nu_1}\cdots\xi^4_{\mu_4\nu_4}K^{\mu_1\cdots\mu_4}K^{\nu_1\cdots\nu_4} \\ &\int_F \frac{d^2\tau}{(\tau_2)^2}\prod_{i=1}^{4}\frac{d^2z_i}{\tau_2}exp\Big\{-\sum_{i<j}k_ik_jG_{ij}\Big\} \end{aligned} \tag{1}$$

where κ is the $10D$ gravitational constant, K is the standard kinematical factor[3] containing four powers of external momenta and G_{ij} is the Green's function on the torus ($T = 1/2\pi\alpha'$):

$$G_{ij}(z_i - z_j) = -\frac{1}{8\tau_2 T}(z_i - z_j - \bar{z}_i + \bar{z}_j)^2 - \frac{1}{2\pi T}\ln\left|\frac{\Theta_1(z_i - z_j|\tau)}{\Theta_1'(0|\tau)}\right| \tag{2}$$

We also use

$$\tau = \tau_1 + i\tau_2, z_i = \eta_i + \tau\sigma_i, 0 \lesssim \eta_i, \sigma_i \lesssim 1 \tag{3}$$

and the integration region F is defined as

$$-\frac{1}{2} \le \tau_1 \le \frac{1}{2}, |\tau| > 1, \tau_2 > 0 \tag{4}$$

The first term in eq.(2) is due to the zero modes of the laplacian on the torus and we shall call it the "zero mode term." For the SST II, the corresponding exponential factor in eq.(1) describes the propagation of the massless states in the loop.[4] The other exponential factor describes the propagation of the massive states.

Let us now study the expansion in α' of the amplitude (1) in some detail. The singularities of this amplitude are due to the integration over $\prod d^2 z_i$ in the region where $|z_i - z_j| \to 0$ and to the integration over τ_2 with $\tau_2 \to \infty$. Since

$$\left|\frac{\Theta_1(z_i - z_j|\tau)}{\Theta'_1(0|\tau)}\right|^{\alpha' k_i k_j} \underset{|z_i - z_j|\approx 0}{\longrightarrow} |z_i - z_j|^{\alpha' k_i k_j} \tag{5}$$

the integration over $\prod d^2 z_i$ gives poles at $\alpha' k_i k_j = 1, 2, \cdots$ which represent the sequence of massive one-particle intermediate states in the one-loop string amplitude. The absence of the massless poles reflects the absence of the one-loop renormalization of the 3-point function with massless external lines. In the low energy sector, $\alpha' k_i k_j \ll 1$, we stay away from the massive poles and therefore we can expand

$$\begin{aligned}
\exp \quad & \left\{ \alpha' \sum_{i<j} k_i k_j \ln |\Theta_1(z_i - z_j|\tau)/\Theta'_1(0|\tau)| \right\} \\
= \; & 1 + \left(\frac{\alpha'}{2}\right)^2 \sum_{i<j} (k_i k_j)^2 \left\{ \sum_{n=1}^{\infty} \frac{1}{n^2} \left[e^{-4\pi n \tau_2 |\sigma_i - \sigma_j|} \right.\right. \\
+ \; & \left.\left. e^{-4\pi n \tau_2 (1 - |\sigma_i - \sigma_j|)} \right] + (q^2 + \bar{q}^2) + 0(q^4) \right\} \\
+ \; & 0\left((\alpha')^3\right) \\
\text{where} \quad q \; & = e^{i\pi\tau} \; .
\end{aligned} \tag{6}$$

The infinite sum in eq. (6) comes from the expansion of the $\ln \Theta_1 \ln \bar{\Theta}_1$ term, the q^2 term from $(\ln \Theta_1)^2$ and the $\bar{q}^2$ from $(\ln \bar{\Theta}_1)$. To get expansion (6) we have used the fact $\sum_i k_i = 0$ and $k_i^2 = 0$ and we have skipped all the η_i - dependent terms which appear as $\exp\{2ni\pi\eta_i\}$ and vanish after integration over η_i. In particular there is no contribution to the amplitude from the term linear in $k_i k_j$. Also, in eq. (6) we specified $\sigma_i - \sigma_j > 0$, refering to the symmetry of the Jacobi Θ-function under the reflection $z_i - z_j \to -(z_i - z_j)$ for the opposite case.

The "zero mass" exponential factor in the amplitude (1) cannot be expanded in α' since the integration over τ_2 gives then term by term divergent result reflecting the branch cuts. Thus, using the expansion (6) the first correction reads (up to the kine-matical and normalization factors):

$$I_1 = \int_i \prod d\eta_i d\sigma_i \int_F \frac{d^2\tau}{\tau_2^2} \exp\{-\pi\tau_2 \sum_{i<j} \alpha' k_i k_j (\sigma_i - \sigma_j)^2\} \tag{7}$$

The integration over τ_2 can be performed by recognizing in eq. (7) the $E_2(b)$ function [6]:

$$E_2(b) = \int_1^\infty \frac{dx}{x^2} e^{-bx} = e^{-b} - bE_1(b) \tag{8}$$

where

$$E_1(b) = -\gamma - \ln b - \sum_{n=1}^{\infty} \frac{(-1)^n b^n}{n \cdot n!} \qquad b\epsilon C, |\arg b| < \pi \tag{9}$$

In our case

$$b = -\sqrt{1-\tau_1^2}\pi\alpha' \Big\{ s[(\sigma_1 - \sigma_2)^2 + (\sigma_3 - \sigma_4)^2] + t[(\sigma_2 \leftrightarrow \sigma_4)] + u[(\sigma_2 \leftrightarrow \sigma_3)] \Big\} \tag{10}$$

where

$$s = 2k_1 k_2, \quad t = 2k_1 k_4 \quad , u = 2k_1 k_3 \tag{11}$$

The obtained correction

$$I_1 = \int \prod_i d\eta_i d\sigma_i \int_0^{1/2} d\tau_1 \frac{2}{\sqrt{1-\tau_1^2}} E_2(b) \tag{12}$$

with b given by eq.(10) should be discussed in some detail. For the terms in the function $E_2(b)$ which are polynomials in b the remaining integrals can be performed order by order in b. The leading (constant) term is the correction calculated by Sakai and Tanii[1] which renormalizes the R^4 term of the effective tree level lagrangian. This leading in the limit $\alpha' \to 0$ one-loop correction is expected to be the same as the leading for $\alpha' \to 0$ contribution of the sum of the $10D$ supergravity diagrams shown in Fig. 1, calculated with a cut-off $1/\alpha'$. Those diagrams are expected to be quadratically divergent and then, with $1/\alpha'$ cut-off, they indeed behave as k^8 *i.e.* as our leading correction. This interpretation is supported by the fact that the correction I_1 has a quadratic in momenta imaginary part.

The next term, linear in b, vanishes after integrating over $d\sigma_i$ due to the condition $s+t+u=0$ and terms of higher order in s,t,u can be easily calculated. It is clear, however, that in addition the correction I_1 has pieces with a complicated branch cut singularity structure (terms of the type $\alpha' k^2 lnk^2$) due to various regions of integration over $d\sigma_i$ of the logarithmic term in $E_2(b)$. A study of the singularity structure of this integral suggests an interpretation of those singularities in terms of the sum of the non-leading (logarithmic) contributions of the field theory diagrams shown in Fig. 1. Note however that the others than the leading one terms in the correction I_1 vanish in the limit $\alpha' \to 0$ and therefore we do not expect this interpretation to go beyond the qualitative level.

Let us now discuss the correction generated by the second term in the expansion (6). The integral over τ_2 can be performed as before. We notice that for the terms proportional to q^2 and $\bar{q}^2$ the result is free of the branch cuts at massless thresholds and the integration over $d\sigma_i$ can also be carried out term by term in powers of α'. This is part of the correction to the effective local 4-graviton vertex generated by the decoupling of heavy modes in the one-loop amplitude. Terms involving $\exp\{-4\pi n\tau_2|\sigma_i - \sigma_j|\}$ can also be integrated over τ_2 by means of eq.(8). It is clear that such terms have branch cuts from the small region of integration over $\prod_k d\sigma_k$ such that $4\pi|\sigma_i - \sigma_j| < \frac{1}{2}\sum_{l,m} \alpha' k_l k_m(\sigma_l - \sigma_m)^2$ and only from this region. This contribution(of the type $(\alpha' k^2)^4 \ln \alpha' k^2$ etc)can be represented by diagrams in Fig. 2. It is a non-local contribution to the effective 4-graviton interaction. The remaining region of integration contributes to the effective local 4-graviton vertex. Altogether, the result for the leading correction to the effective 4-graviton vertex generated by the decoupling of heavy modes in the loop (Fig. 3) reads:

$$\frac{\alpha'\kappa^4}{2^9\pi^6}\xi^1_{\mu_1\nu_1} \quad \cdots \quad \xi^4_{\mu_4,\nu_4} K^{\mu_1\cdots\mu_4} K^{\nu_1\cdots\nu_4}(s^2+t^2+u^2) \cdot \left(\frac{\xi(3)}{2\pi}(\frac{1}{3} - \frac{1}{4}\ln 3)\right) \tag{13}$$

Contrary to the Sakai-Tanii term, in the spirit of the decoupling philosophy, this is a correction to the tree level effective lagrangian

$$\Delta\mathcal{L}_{eff} = \frac{1}{2\kappa^2}\frac{\alpha'^3}{8}\left[3\frac{\alpha'^2 g^2}{2^6\pi^6}\frac{1}{2\pi}\xi(3)(\frac{1}{3} - \frac{1}{4}\ln 3)\right] \cdot t^{\mu_1\cdots\mu_8} t^{\nu_1\cdots\nu_8}(D_\rho R_{\mu_1\mu_2\nu_1\nu_2})(D^\rho R_{\mu_3\mu_4\nu_3\nu_4})(D_\sigma R_{\mu_5\mu_6\nu_5\nu_6}) \cdot (D^\sigma R_{\mu_7\mu_8\nu_7\nu_8}) \tag{14}$$

(There is only one independent tensor structure of the desired form). The correction (14) has the same tensor structure as $(\alpha' k^2)^5$ correction obtained from the tree level amplitude.

This analysis can be extended to higher orders in the expansion (6). Various terms can be classified by the number of factors $\exp\{-4\pi n\tau_2|\sigma_i - \sigma_j|\}$ they contain. For instance with two such factors one gets contribution corresponding to the diagram in Fig. 4 and of course one corresponding to the diagram in Fig. 3. With four such factors there are no branch cuts at massless thresholds and one only gets a contribution to the effective local vertex (Fig.3).

III. One-Loop Corrections to the Four-Graviton Interaction in the Heterotic String Theory

Using the path integral approach one gets the following amplitude for the 4-graviton scattering[6] (with the same as before conventions for the measures)

$$\begin{aligned} A(1,2,3,4) &= \frac{\kappa^4}{2^7}\int_F \frac{d^2\tau}{4\pi\tau_2^2}(2\pi\tau_2)^{-4}(\eta(\tau))^{-24}(2\pi\alpha')^{-5} \\ &\cdot \frac{1}{4}\Big[\sum_{(\alpha,\beta)\neq(1,1)}(\Theta[^\alpha_\beta](0|\tau))^8\Big]^2\prod_{i=1}^{4}\int d^2z_i\zeta_1^{\mu_1\nu_1}\cdots\zeta_4^{\mu_4\nu_4} \\ &\cdot (\text{bosons})_{\nu_1\cdots\mu_4} \qquad (\text{fermions})_{\mu_1\cdots\mu_4} \end{aligned} \tag{15}$$

where

$$(\text{fermions})_{\mu_1\cdots\mu_4} = 2^5 t_{\sigma_1\mu_1\cdots\sigma_4\mu_4}k_1^{\sigma_1}\cdots k_4^{\sigma_4}$$

$$\begin{aligned} (\text{bosons})^{\nu_1\cdots\nu_4} &= \Big\{\Big[(\partial_1\partial_2G_{12})(\partial_3\partial_4G_{34}) + (2\leftrightarrow3) + (2\leftrightarrow4)\Big] \qquad (16) \\ &- \sum_{i,j=1}^{4} k_ik_j\Big[(\partial_1G_{1i})(\partial_2G_{2j})(\partial_3\partial_4G_{34}) + (2\leftrightarrow3) + (2\leftrightarrow4) \\ &+ (1\leftrightarrow3) + (1\leftrightarrow4) + (1\leftrightarrow3,\quad 2\leftrightarrow4)\Big] \\ &+ \sum_{i,j,k,l=1}^{4} k_ik_jk_kk_l(\partial_1G_{1i})(\partial_2G_{2j})(\partial_3G_{3k})(\partial_4G_{4l})\Big\} \\ & e^{-\sum_{i<j}k_ik_jG_{ij}} \end{aligned} \tag{17}$$

Here G_{12} implicitly carries the factor $\delta^{\nu_1\nu_2}$ and $k_iG_{1i} \equiv k_i^{\nu_1}G_{1i}$($G_{ij}$ is defined in eq. (2)).

We integrate by parts all double derivatives and get (all indices are now explicit):

$$\prod_i \int d^2z_i(\text{bosons})^{\nu_1\cdots\nu_4} = \left\{ \left[\prod_i \int d^2z_i \ (\partial_1 G_{12})(\partial_2 \ G_{21})(\partial_3 G_{34})(\partial_4 G_{43})\right] \right.$$

$$\cdot \ t_1^{\sigma_1\nu_1\cdots\sigma_4\nu_4} + \left.\left[\prod_i \int d^2z_i(\partial_1 G_{12})(\partial_2 G_{23})(\partial_3 G_{34})(\partial_4 G_{14})\right] t_2^{\sigma_1\nu_1\cdots\sigma_4\nu_4}\right\}$$

$$\cdot \ k_1^{\sigma_1}\cdots k_4^{\sigma_4} e^{-\sum_{i<j} k_ik_jG_{ij}} \tag{18}$$

We have used the periodicity of G_{ij} : $G(\nu,\tau) = G(\nu+1,\tau) = G(\nu+\tau,\tau)$ and the fact that in the limit $z \to z\prime$ singularities of $\partial_z G(z-z\prime,\tau)$ and $\partial_z^2 G(z-z\prime,\tau)$ are integrable to zero.[1,2] The tensors t_1 and t_2 are defined in eq. (4.A.23) of ref.[3] where

$$t = a_1\epsilon + a_2t_1 + a_3t_2 \tag{19}$$

Due to the freedom of relabelling particles it is sufficient to calculate those two integrals only.

Now we can proceed with the analysis similar to that of the previous section, although technically more involved. As before the leading one-loop correction is obtained by effectively replacing the exponential factor in eq.(18) by 1. We are left with two integrals over $\prod d^2z_i(d^2z_i = 2d(Rez_i)d(Imz_i))$

$$I_1 = \left[\int d^2z_1 d^2z_2(\partial_1 G_{12})(\partial_2 G_{21})\right]^2 \tag{20}$$

$$I_2 = \int \prod_i d^2z_1(\partial_1 G_{12})(\partial_2 G_{23})(\partial_3 G_{34})(\partial_4 G_{41}) \tag{21}$$

We calculate them in a pedastrian way by expanding ($z = \eta + \tau\sigma, \nu = z - z'$)

$$\partial_z G(z,z') = -\frac{i(\sigma-\sigma')}{2T} + \frac{i}{4T} + \frac{i}{2T}[e^{2\pi i\nu} + e^{4\pi i\nu} + \ldots]$$

$$
\begin{aligned}
&+ \quad \frac{i}{2T} q^2(1+q^2)(e^{2\pi i\nu} - e^{2\pi i\nu}) \\
&+ \quad \frac{i}{2T} q^4(e^{4\pi i\nu} - e^{-4\pi i\nu}) + 0(q^6) \qquad (22)
\end{aligned}
$$

then integrating (20) and (21) term by term (the η dependent terms are of the form $\exp(2\pi in\eta)$ and do not contribute to the integrals) and recognizing that the result should be modular covariant. This way we get[7]

$$
I_1 = \left[\frac{\tau_2^2}{12T^2}(1 - 24q^2 - 72q^4 + ...)\right]^2 = \left(\frac{\tau_2}{2\pi T}\right)^4 G_2^2(q) \qquad (23)
$$

$$
I_2 = \frac{\tau_2^4}{T^4 720}(1 + 240q^2 + 2160q^4 + ...) = \left(\frac{\tau_2}{2\pi T}\right)^4 G_4(q) \qquad (24)
$$

Here $G_2(q)$ and $G_4(q)$ are modular forms (see, for instance Appendix 6 of ref. [4]and ref. [8]).

A comment is in order here. It is well known that $G_2(\tau)$ has modular anomaly: $G_2(\tau) = \hat{G}_2(\tau, \bar{\tau}) + \frac{\pi}{\tau_2}$ where $\hat{G}_2$ is modular covariant. Thus, G_2 in eq. (23) should in fact be replaced by $\hat{G}_2$ for a modular covariant result. This reflects the fact that the propagator is singular at $\nu = 0$ and proceeding more carefully one should use properly regularized derivative rather than expansion (22)[7].

Modular covariance of the integrals I_1 and I_2 is crucial for the remaining integration over τ[2,10] but the term π/τ_2 which cancels the anomaly of G_2 does not contribute to the final result explicitly. The result reads:

$$
\int \frac{d^2\tau}{4\pi\tau_2^2}(2\pi\tau_2)^{-4}\eta(\tau)^{-24}\frac{1}{4}\Big[\sum_i \Theta_i^8\Big]^2 I_{1,2} = \frac{\alpha'^4}{6} \qquad (25)
$$

Using (15),(18) and the relation $t_2 = 2t + t_1 + \epsilon$–term we finally get the following one-loop correction

$$
A^c(1,2,3,4) = 2\kappa^2 \frac{g^2\alpha'^3}{2^6 3\pi^5} \varsigma_1^{\mu_1\nu_1} \cdots \varsigma_4^{\mu_4\nu_4} t_{\sigma_1\mu_1\cdots\sigma_4\mu_4} \cdot
$$

$$\cdot \quad k_1^{\sigma_1} \cdots k_4^{\sigma_4} k_1^{\delta_1} \cdots k_4^{\delta_4} \left[t_{\delta_1 \nu_1 \cdots \delta_4 \nu_4} + t^1_{\delta_1 \nu_1 \cdots \delta_4 \nu_4} \right] \tag{26}$$

The corresponding renormalization of the tree level[11] effective lagrangian reads:

$$\begin{aligned} L = L_o + L_1 &= \frac{1}{2\kappa^2} \left\{ \left[-R + 0(R^2) + \frac{(\alpha')^3}{2^4} (2\xi(3) Y + Y^1) \right] \right. \\ &+ \left. \frac{(\alpha')^3 g^2}{2^7 \cdot 3\pi^5} (Y + Y^1) \right\} \end{aligned} \tag{27}$$

where

$$Y = t_{\sigma_1 \mu_1 \cdots \sigma_4 \mu_4} t_{\rho_1 \nu_1 \cdots \rho_4 \nu_4} R^{\sigma_1 \mu_1 \rho_1 \nu_1} \cdots R^{\sigma_4 \mu_4 \rho_4 \nu_4} \tag{28}$$

$$Y^1 = t_{\sigma_1 \mu_1 \cdots \sigma_4 \mu_4} t_{\rho_1 \nu_1 \cdots \rho_4 \nu_4} R^{\sigma_1 \mu_1 \rho_1 \nu_1} \cdots R^{\sigma_4 \mu_4 \rho_4 \nu_4} \tag{29}$$

Next, we can proceed to the calculation of non-local corrections with massless branch cuts (which are of higher order in α') generated when the total contribution corresponding to the first term in expansion (6) is taken into account. The analysis is however much more complicated. Still, one can classify various contributions by assigning to them field theory diagrams with the same branch cut singularity structure. Our analysis shows, however, that the branch cut singularities of the heterotic string one-loop amplitude are not the same as the singularities of the field theory one-loop diagrams generated by the tree level lagrangian $R + R^2$. For instance the $k^{12} \ln k^2$ singularity corresponding to the diagrams in Fig. 5 seems to be absent. This may not be so surprising since such terms are non-leading in the limit $\alpha' \to 0$.

IV. Summary

We have explicitly sorted out various corrections to the four-graviton interaction generated by the one-loop amplitudes in the SST II and the heterotic string theory.

As expected they are of two types. Some simply renormalize the local vertices already present at the tree level. There in fact belongs the leading as $\alpha' \to 0$ 1-loop correction (in both string theories). This contribution has been first calculated in ref.[1] for the SST and in the field theoretical language it corresponds to the leading contribution of the one-loop supergravity diagrams in Fig. 1 calculated with a cut-off $1/\alpha\prime$. In this paper the analogous correction is also explicitly obtained for the heterotic string.

Among non-leading local corrections are also those which are generated by the decoupling of heavy modes propagating in the loop, as in Fig. 3. Within the framework of the general decoupling philosophy such corrections should in fact be included into the tree level lagrangian. We calculate explicitly the leading correction of this type in the SST II.

Of course, the one-loop amplitudes generate also non-local corrections with branch cut singularities at massless thresholds. It is not easy to present them in a compact form but they can be classified by assigning to them field theory diagrams with the some branch cut singularity structure.

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